AN EFFICIENT LINEAR STATISTICAL FET MODEL

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ABSTRACT

The commonly used FET model is examined and found to be at best a difficult structure for modeling a FET's performance statistics. A simpler linear statistical model based on Principal Component Analysis is proposed which results in uncorrelated model parameters. An example using actual measured GaAs FET data uses just 13 uncorrelated random variables to model the FET's performance statistics from 1 to 11 GHz.

INTRODUCTION

Statistical circuit design methods are presently being developed for the design of high yield microwave circuits. This emphasis on circuit yield is due in part to the need for high yield monolithic microwave integrated circuits. This effort is accelerating the development of tools for the statistical design and simulation of microwave circuits. Presently several microwave CAD packages offer some sort of statistical yield "optimizer" for the design of high yield microwave circuits.

One issue in statistical circuit design is the statistical modeling of the microwave components. This area has not been discussed in the literature. The purpose of this paper is to analyze the present methods of FET statistical modeling, point out difficulties with the present approach, and propose a new statistical FET model which has better statistical properties. Because this might be a new concept to some, first we will define what is meant by a statistical model. To be specific we will consider the FET as an example.

STATISTICAL FET MODELS

The goal of a statistical FET model is to accurately describe the statistics of the FET electrical properties using a small number of parameters. In general the description needs to accurately describe the FET's statistical behavior over frequency. The main electrical properties that are measured by the FET manufacturer are the FET's 2-port S-paramaters. The S-parameters are in reality random variables with a joint distribution function. Simulation using a valid statistical FET model will create simulated S-parameters with the same statistics as the measured S-parameters.

The present approach to FET modeling is to extract from the S-parameter data over frequency a set of FET model parameters (c_{gs}, r_d, g_m, ect.) which "fit" the data. The "fit" is accomplished by determining a set of FET model parameters for each measured set of S-parameters and averaging the FET model parameters to give the nominal FET parametric description.

The procedure presently being used to statistically model and simulate the FET is to record the statistics of the FET model parameters and attempt to recreate the S-paramter statistics by using the measured FET model parameter statistics in the simulation. This paper will show that this is a statistically difficult problem at best. We will propose a simpler and more effecient statistical model.

Nonlinear Transformations on Random Variables

The present modeling method starts with a set of jointly distributed random variables (S-parameters) and maps them using a nonlinear transformation into another set of random variables (FET model parameters).

S <==> M.

In general to recreate the S-parameter statistics using the model variables two criteria must be met:

1) the mapping needs to be 1 to 1
2) the joint distribution function for the M's must be known.

In practice it is difficult to determine and record the full joint distribution of the model parameters. Usually only the marginal densities and the correlations and cross correlations are measured and recorded. Unless the data is Gaussian, this is not sufficient to recreate the S-parameter joint distributions. A simple example illustrates this situation.

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Example

Consider two independent random variables S1 and S2 each uniformly distributed [0,1]. We will nonlinearly transform these variables into a new set by; M1 = S1/S2, and M2 = S2. M1 and M2 are not independent and furthermore M1 is not uniform. M1 lies on the interval [0,∞]. Next we attempt to recreate the original S1 and S2 by simulating M1 and M2 according to their marginal densities and correlations and mapping M1 and M2 through the inverse transformation. However this is not successful because the recreated S1, call it S1', lies on the interval [0,∞], because the inverse mapping is S1' = M1 * M2. This illustrates the FET model problem if one considers S1 and S2 to be the S-parameters and M1 and M2 to be the model parameters. To properly recreate S1 in this example we would need the full joint distribution function for M1 and M2, not just the marginal densities and their correlations. The same holds true in general for the FET model parameters.

This example illustrates the problem with the statistical simulation of the FET electrical performance by using the standard FET model and the model parameters; in general the entire joint density function for the FET parameters must be known.

A STATISTICALLY EFFICIENT FET MODEL

The goal is to accurately record the statistical behavior of the FET S-parameters with a small number of parameters while using a simple model. The following approach, based on principal component analysis accomplishes both of these.

Principal Component Analysis

A principal component analysis (PCA) of a set of m zero mean, unit variance random variables (S1,S2,...,Sm) creates m new uncorrelated random variables, the principal components (PC), K1, K2,...,Km, with each PC being a linear combination of the original variables, that is;

$$\begin{aligned} K1 &= b11S1 + b12S2 + \ldots + b1mSm \\ K2 &= b21S1 + b22S2 + \ldots + b2mSm \\ &\vdots \\ Km &= bm1S1 + bm2S2 + \ldots + bmmSm \end{aligned}$$

or in matrix form K = BS [1]. The coefficients for K1 are chosen to make its variance as large as possible. The coefficients for K2 are chosen to make its variance as large as possible, subject to the restriction that K1 (whose variance has already been maximized) be uncorrelated with K2. This continues in general for all the K's. The important thing to note here is the statistical description of the K's is simplified because they are uncorrelated. We propose to use these K's as the statistical model parameters, and use the linear model;

$$S = B^{-1} K$$

as the statistical FET model.

An important property of the PCA is the ability to reduce the number of K's in the model by identifying the K's which are statistically insignificant. Essentially the number of significant K's needed in the model description represents the number of independent degrees of freedom present in the S population. The example which follows determines that 13 uncorrelated principal components are needed to represent the S-parameter statistics for a 0.5 micron GaAs FET from 1 to 11 GHz. The reduced model then becomes the 13 columns of the B^{-1} matrix that are associated with the 13 significant PC's.

Example

This example starts with actual measured S-parameter data from 90 GaAs FETS from 1 to 11 GHz. We then perform the principal component analysis, and compare the marginal densities and the correlations of the measured and simulated data over the entire frequency range.

The FETS were fabricated during January 1987 to June 1987 with TriQuint Semiconductor Inc.'s standard process. The 0.5 X 300 um FETS are described in [2]. Data was taken on 90 FETs. Because this is not a large sampling, this example probably represents a more difficult modeling problem than if the number of FETs measured were larger. The FETS were measured at 1, 3.5, 6, 8.5 and 11 GHz at Id = Idss. The S-parameter data was put into real and imaginary form and since there are four S-parameters, there are 8 data points for each frequency. The data for each FET was arranged into a vector of 40 elements, 8 elements for each frequency times the 5 frequencies at which the FETS were measured. Therefore the S-parameter data consisted of 90 vectors of length 40. Each random variable was normalized to zero mean and unit variance. This data represents samples from 40 random variables, and consequently the joint distribution function has dimension of 40. The marginal densities for the 40 S-parameters are not in general Gaussian or "bell-shaped". Figure 1 shows the marginal density functions for the real part of S11 at the five frequencies.

The S-parameter data is also highly correlated. In general a 40 x 40 correlation matrix describes the S-parameter correlations. Samples of this matrix are shown in Tables 1 and 2.

A Principal component analysis was then initiated on this data using the S.A.S. statistical analysis package using the Quartimax rotation [3]. This analysis then gave a 40 X 40 coefficient matrix, B.

1.00	-0.59	-0.92	-0.29	-0.59	-0.02	0.19	-0.44
-0.59	1.00	0.60	-0.26	0.47	0.55	-0.04	0.72
-0.92	0.60	1.00	0.01	0.57	-0.02	-0.24	0.44
-0.29	-0.26	0.01	1.00	-0.10	-0.41	0.32	-0.28
-0.59	0.47	0.57	-0.10	1.00	0.58	-0.80	0.06
-0.02	0.55	-0.02	-0.41	0.58	1.00	-0.44	0.21
0.19	-0.04	-0.24	0.32	-0.80	-0.44	1.00	0.24
-0.44	0.72	0.44	-0.28	0.06	0.21	0.24	1.00

TABLE 1. The 8X8 correlation matrix of the measured S-parameters at 3 GHz. The ordering is real then imaginary parts of S11, S12, S21, S22.

1.00	0.97	0.95	0.86	0.76
0.97	1.00	0.99	0.94	0.85
0.95	0.99	1.00	0.97	0.90
0.86	0.94	0.97	1.00	0.98
0.76	0.85	0.90	0.98	1.00

TABLE 2. The 5X5 correlation matrix of measured real part of S11 at 1, 3.5, 6, 8.5, and 11 GHz presented in this order.

The S.A.S. analysis indicated there were 13 statistically significant principal components. We then found the appropriate 13 columns of B^{-1} to represent the statistical model. The reduced 40x13 B^{-1} matrix is partially shown below.

$B^{-1'}$ =

-0.962	0.008	-0.139	0.117	-0.004	. . .	0.004
-0.973	0.081	-0.001	0.077	-0.001	. . .	-0.004
-0.575	-0.150	0.019	-0.787	-0.015	. . .	-0.002
.	.	.	.	.		.
.	.	.	.	.		.
.	.	.	.	.		.
0.687	-0.632	0.295	0.042	0.001	. . .	-0.021

To test the model we simulated the 13 uncorrelated PC's according to their marginal distributions, which were obtained using the measured S-paramter data and the linear model K = BS. A sample set of 500 points was generated. The simulated S-parameters were then created from the reduced order model $S = B^{-1'}K$, where $B^{-1'}$ is the 40x13 reduced parameter matrix.

A comparison was then made between the measured S-parameter marginal densities and correlations and the simulated S-parameter marginal densities and correlations. The data matched remarkably well considering the small population of measured data that was used. Figure 2 shows the simulated marginal densities of the real part of S11 at 1, 3.5, 6, 8.5, and 11 GHz. Table 3 shows the correlation matrix of the simulated S-parameters at 3 GHz and Table 4 shows the 5X5 correlation matrix of the simulated real part of S11 at 1, 3.5, 6, 8.5, and 11 GHz. In general the measured and simulated data matched well as is evidenced by a comparison of Figures 1 and 2 and Tables 1,2,3, and 4.

1.00	-0.59	-0.93	-0.35	-0.60	0.01	0.16	-0.42
-0.59	1.00	0.60	-0.20	0.45	0.54	0.02	0.70
-0.93	0.60	1.00	0.10	0.57	-0.05	-0.19	0.40
-0.35	-0.20	0.10	1.00	-0.05	-0.43	0.28	-0.25
-0.60	0.45	0.57	-0.05	1.00	0.57	-0.80	0.03
0.01	0.54	-0.05	-0.43	0.57	1.00	-0.42	0.21
0.16	0.02	-0.19	0.28	-0.80	-0.42	1.00	0.28
-0.42	0.70	0.40	-0.25	0.03	0.21	0.28	1.00

TABLE 3. The 8X8 correlation matrix of the simulated S-parameters at 3 GHz. The ordering is real and imaginary parts of S11, S12, S21, S22.

1.00	0.97	0.95	0.87	0.77
0.97	1.00	1.00	0.94	0.86
0.95	1.00	1.00	0.97	0.90
0.87	0.94	0.97	1.00	0.98
0.77	0.86	0.90	0.98	1.00

TABLE 4. The 5X5 correlation matrix of simulated real part of S11 at 1, 3.5, 6, 8.5, and 11 GHz in this order.

CONCLUSIONS

Now that statistical circuit design is being implemented by the microwave industry, it is time to carefully study the models used in microwave CAD to see if they are statistically valid. This paper shows that the present FET model using model parameter statistics is at best difficult because the model parameters are correlated and in general the entire joint distribution function of the parameters must be used in the simulation.

A simpler linear statistical model based on principal component analysis is proposed and a simple example illustrates its potential. This example determines 13 uncorrelated random parameters which statistically describe the FET from 1 to 11 GHz, using only the parameter marginal distributions.

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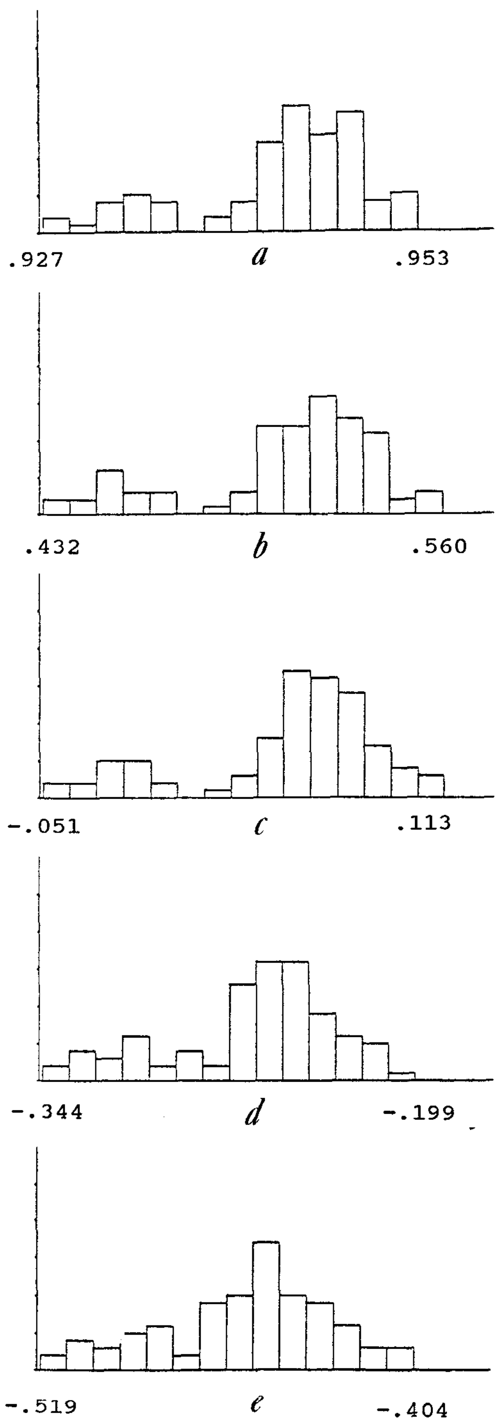

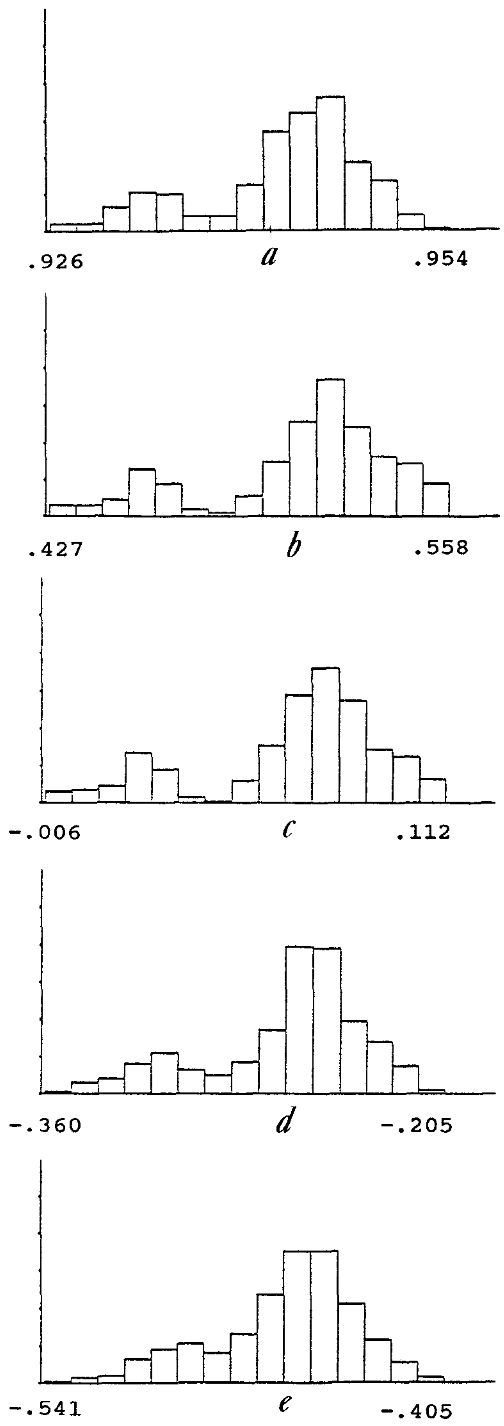

FIGURE 1. Marginal Densities For The Measured Real Part of S11 at a) 1 GHz, b) 3.5 GHz, c) 6 GHz, d) 8.5 GHz, and e) 11GHz.

FIGURE 2. Marginal Densities For The Simulated Real Part of S11 at a) 1 GHz, b) 3.5 GHz, c) 6 GHz, d)8.5 GHz, and e) 11GHz.